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Elastic Scattering of Alpha Particles by O^{18}

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The differential elastic scattering cross section of alpha particles by O^{18} has been measured for alpha-particle energies between 2.4 and 3.5 MeV and for $\theta_{c.m.}=90.0°$, 125.3°, 140.7°, 152.3°, and 164.4° in order to determine the level parameters of excited states in Ne^{22}. The resonances were analyzed using the single-level and two-level approximation to determine reduced widths and resonance energies of the levels. Scattering anomalies were observed at bombarding alpha-particle energies of 2.48, 2.56, 2.72, 2.90, 3.17, and 3.33 MeV which correspond to excited states of Ne^{22} with the following energies, angular momenta, and parities: 11.70 MeV, $J^\pi=2^+$; 11.76 MeV, $J^\pi=1^-$; 11.89 MeV, $J^\pi=1^-$; 12.04 MeV, $J^\pi=0^+$; 12.26 MeV, $J^\pi=0^+$; 12.39 MeV, $J^\pi=2^+$. For bombarding alpha-particle energies in the vicinity of 3.5 MeV, it was found necessary to include the effects of two levels above 3.5 MeV to fit the scattering data: 12.57 MeV, $J^\pi=1^-$ and 12.61 MeV, $J^\pi=1^-$. None of the levels observed have reduced widths greater than 9% of the Wigner limit.

INTRODUCTION

THE bombardment of O^{18} by alpha particles of energy 2.40 to 3.50 MeV produces excitation energies of 11.63 to 12.53 MeV in the compound nucleus Ne^{22}. Bair and Willard[1] have recently studied excited states in Ne^{22} in this energy region by means of the $O^{18}(\alpha,n)Ne^{21}$ reaction. Their measurements, which yielded total cross sections and energies of the excited states, were unable to unambiguously assign spins and parities and reduced widths of these states. The present experiment, $O^{18}(\alpha,\alpha)O^{18}$, was undertaken as an attempt to determine these parameters.

Since both the alpha particle and the O^{18} nucleus have zero spin, it is only possible to form excited states of odd angular momentum and odd parity, or of even angular momentum and even parity. By choosing the center-of-mass scattering angles at the zeros of the Legendre polynomials, one can assign spins and parities simply by the presence or absence of a resonance effect at the various angles. From the $O^{18}(\alpha,n)Ne^{21}$ data one can make first estimates of the reduced widths which can then be subsequently adjusted to fit the elastic scattering data.

EXPERIMENTAL

The $(He^4)^+$ beam of the ORNL 5.5-MV Van de Graaff was stripped to $(He^4)^{++}$ and then magnetically analyzed by bending through 90° prior to striking thin targets of O^{18}. The targets (<2 keV for a 2.40-MeV alphas) were prepared[2] by oxidizing W with water vapor enriched to greater than 97% O^{18} and then evaporating the W_4O_{11} thus obtained onto 25 $\mu g/cm^2$ carbon backings. The targets were made sufficiently thin to study the narrow resonances observed in the $O^{18}(\alpha,n)Ne^{21}$ experiment, and were found to withstand many hours of bombardment with ≈ 0.4-μA beam currents. The incident alpha-particle energy was determined by calibrating the 90° analyzing magnet by means of the 1880.4-keV threshold[3] in the $Li^7(p,n)Be^7$ reaction, and then adjusting the calibration constant with the known ratio of alpha particle to proton mass. The calibration constant is estimated to be accurate to $\pm 0.2\%$.

Experimental data were taken on two separate occasions. During the first series of measurements the elastically scattered alpha particles were observed at $\theta_{c.m.}=90.0°$, 125.3°, 140.7°, and 152.3° by means of a scintillation counter placed at the focal plane of a 60°, double-focusing, uniform-field, reaction-product magnet of momentum resolution $p/\Delta p=500$. The angles 90.0°, 125.3°, and 140.7° correspond, respectively, to the zeros of Legendre polynomials of odd order, order two, and order three. The angle 152.3° was the maximum scattering angle obtainable with the reaction-product magnet; at this angle all the resonances in the scattering data should be present. The data were taken with 6% counting statistics at 5- to 12-keV energy intervals over the entire energy region except in the vicinity of the first narrow resonance where data were taken at smaller intervals. Measurements were made with the same statistical accuracy several months later at $\theta_{c.m.}=140.7°$ and 164.4° with a silicon surface barrier detector instead of the reaction-product magnet. The movable surface barrier detector was mounted directly in the scattering chamber. In addition, the data at $\theta_{c.m.}=90.0°$ were repeated with the magnet. The agreement between the two sets of data was satisfactory.

When the reaction-product magnet is used, not all of the scattered alpha particles reach the detector at the focal plane of the magnet, since, for scattered alpha particles of energies less than 2 MeV, some of the alpha particles emerge from the target in charge states He^0 or He^+. The true scattering yield may be obtained by multiplying the observed yield by the fraction $(He^0+He^++He^{++})/He^{++}$. This ratio is obtained from

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[1] J. K. Bair and H. B. Willard, Phys. Rev. **128**, 299 (1962).

[2] A. H. F. Muggleton and F. A. Howe, Nucl. Instr. and Methods **12**, 192 (1961).

[3] J. B. Marion, Second International Conference on Nuclidic Masses, Vienna, July 1963 (unpublished).

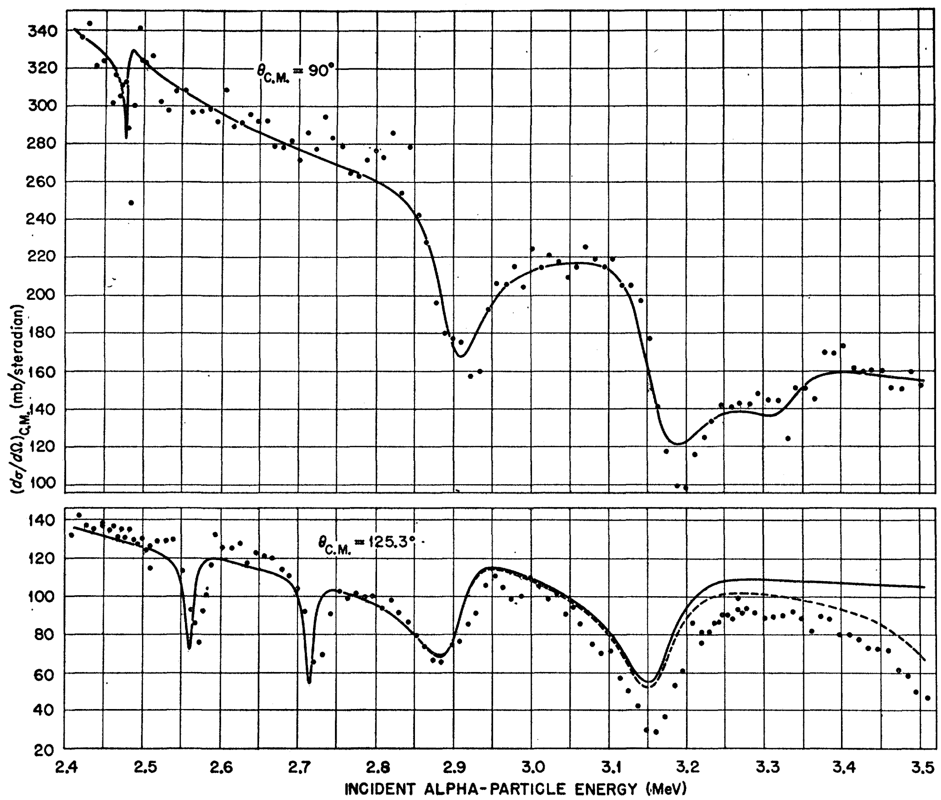

FIG. 1. The differential cross sections for elastic scattering of alpha particles by O^{18} at the center-of-mass angles of 90° and 125.3°.

tabulated charge equilibrium fractions,[4] and was used to determine the true scattering yield for the magnet data. After making the charge-exchange corrections, the data were then normalized at 2.40 MeV to the Rutherford-plus-hard-sphere scattering cross sections.

RESULTS

A. Spin and Parity Assignment

The normalized experimental data are plotted in Figs. 1 and 2, and the parameters obtained by analysis of these data are tabulated in Table I. The spins and parities of the levels are readily assigned on the following basis: (1) A $J^\pi=0^+$ resonance ($l=0$ partial wave) appears at all angles; the resonances at $E_\alpha^{\mathrm{Lab}}=2.90$ and 3.17 MeV fall into this category; (2) A $J^\pi=1^-$ resonance ($l=1$ partial wave) appears at all angles except $\theta_{\mathrm{c.m.}}=90°$; the resonances at $E_\alpha^{\mathrm{Lab}}=2.56$ and 2.72 MeV fit this description; (3) A $J^\pi=2^+$ resonance ($l=2$ partial wave) appears at all angles except $\theta_{\mathrm{c.m.}}=125.3°$; the anomalies at $E_\alpha^{\mathrm{Lab}}=2.48$ and 3.33 MeV fit this description.

The assignment of 1^- for level 3 at 2.72 MeV (11.89-MeV excitation in Ne^{22}) is seen to differ from the assignment of 2^+ as reported by Deuchars and Dandy.[5] The presence of a resonance effect at 125.3° and the absence of a resonance effect at 90° as indicated at 2.72 MeV in Fig. 1 is inconsistent with a 2^+ assignment, and it is our conclusion that their assignment is incorrect.

[4] J. B. Marion, *1960 Nuclear Data Tables* (National Research Council, Washington, D. C., 1960), Part 3, p. 26.

[5] W. M. Deuchars and D. Dandy, Proc. Phys. Soc. (London) **77**, 1197 (1961).

B. Data Analysis

In an $O^{18}+\alpha$ experiment with alpha-particle bombarding energies of 2.40 to 3.50 MeV, the only energetically possible exit channels are those for the emission of a gamma ray, a neutron, or an alpha particle. (Γ_α to the 1.98-MeV level of O^{18} is energetically possible above 2.40 MeV, but the Coulomb barrier inhibits the inelastic cross section.) The total width of each resonance must therefore be $\Gamma=\Gamma_\alpha+\Gamma_n+\Gamma_\gamma$. We have assumed throughout the analysis that the partial width for capture, Γ_γ, is negligible compared to Γ_α and Γ_n.

Blatt and Biedenharn[6] give an expression in the one-level approximation for the elastic scattering of spinless particles by spinless nuclei which applies to the case where more than one exit channel is open. Preliminary attempts at fitting the present data with their formula gave fair qualitative agreement between experiment and theory, but it was soon apparent that levels of the same spin and parity were interfering with one another.

[6] J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24**, 258 (1952).

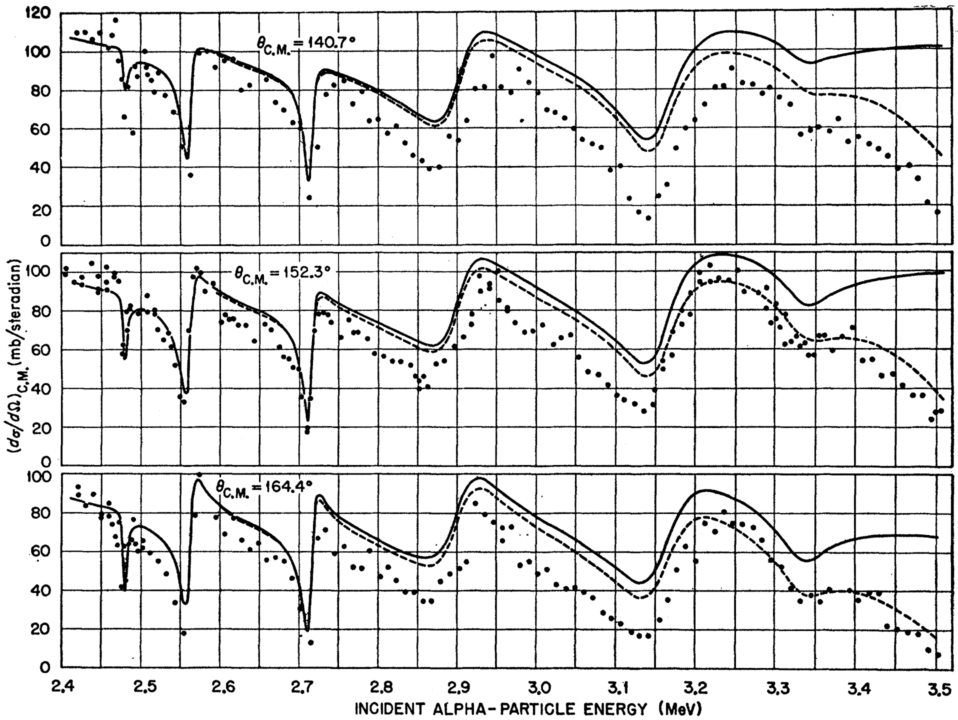

FIG. 2. The differential cross sections for elastic scattering of alpha particles by O^{18} at center-of-mass angles of 140.7°, 152.3°, and 164.4°.

Lane and Thomas[7] derive an expression for the total reaction cross section of two interfering levels of the same spin and parity; they also give considerable background and discussion for differential elastic scattering cross sections but do not formally derive this cross section in the two-level approximation. Yagi[8] has recently applied in a very lucid manner the results of the Lane and Thomas article to the case of two interfering levels of the same spin and parity for the elastic scattering of protons by O^{18}. By following the analysis given in the two papers, one can obtain an expression for the differential elastic scattering cross section of spinless particles by spinless nuclei when two levels of the same spin and parity are interfering and when more than one exit channel is open. We state without proof the result in the notation of Lane and Thomas.

Two-Level Formula

$$\frac{d\sigma(\theta)}{d\Omega}=\lambda\!\!\!^{-}_{\alpha}{}^{2}\Bigg[-(\pi)^{1/2}C(\theta)+\frac{i}{2}\sum_{l'=0}^{\infty}(2l'+1)P_{l'}(\cos\theta)\left[e^{2i\omega_{l'}}-e^{2i(\omega_{l'}-\varphi_{l'})}\right]$$
$$+\sum_{\substack{l=0\\ \text{(sum to include}\\ \text{all resonances)}}}^{2}\frac{(2l+1)P_l(\cos\theta)\rho_\alpha e^{2i(\omega_l-\varphi_l)}}{F_l^2+G_l^2}\left[\gamma_{\alpha 1l}{}^2A_{11}+\gamma_{\alpha 2l}{}^2A_{22}+2(\gamma_{\alpha 1l}{}^2\gamma_{\alpha 2l}{}^2)^{1/2}A_{12}\right]\Bigg]^2, \quad (1)$$

where

$$C(\theta)=\frac{1}{(\pi)^{1/2}}\frac{\eta_\alpha}{2}\csc^2\left(\frac{\theta}{2}\right)\exp\{i\eta_\alpha\ln\csc^2\theta/2\}, \quad (2)$$

θ is the center-of-mass scattering angle, and $P_l(\cos\theta)$ is the Legendre polynomial of order l. The quantities $\lambda\!\!\!^{-}_\alpha=1/k_\alpha$ and η_α are related to the reduced mass μ and relative velocity v of the scattering pair by

$$\lambda\!\!\!^{-}_\alpha=1/k_\alpha=\hbar/\mu v, \quad (3)$$

$$\eta_\alpha=Z_1Z_2e^2/\hbar v. \quad (4)$$

[7] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).
[8] K. Yagi, J. Phys. Soc. Japan **71**, 604 (1962).

TABLE I. Parameters of the energy levels of Ne^{22} determined by analysis of the present experiment.

Level	E_{res}(Lab) (MeV)	E_{exc} (MeV)	J^π	Γ(Lab) (keV)	Γ_α(c.m.) (keV)	γ_α^2(c.m.) (keV)	θ_α^2 [b] (%)	Γ_n(c.m.) (keV)	γ_n^2(c.m.) (keV)	θ_n^2 [c] (%)
1	2.48	11.70	2^+	6.0	0.5	8.2	1.7	4.4	1.6	0.08
2	2.56	11.76	1^-	14	3.1	15	3.1	8.4	4.4	0.21
3	2.72	11.89	1^-	13	3.6	11	2.3	7.1	3.4	0.17
4	2.90	12.04	0^+	70	29	41	8.5	29	34	1.70
5	3.17	12.26	0^+	90	47	43	8.9	26	26	1.30
6	3.33	12.39	2^+	81	4.8	9.0	1.9	61	18	0.87
7[a]	(3.55)	(12.57)	(1^-)	(32)	(12)	(10)	(2.1)	(14)	(5.0)	(0.24)
8[a]	(3.60)	(12.61)	(1^-)	(115)	(46)	(35)	(7.3)	(49)	(18)	(0.85)

[a] The effects of these levels were observed in Fig. 1 in the vicinity of 3.40 to 3.50 MeV. The parameters are enclosed in parentheses since the actual levels were not observed in the scattering data.

[b] $\theta_\alpha^2=\gamma_\alpha^2\left[\frac{2M\alpha M_{O^{18}}}{3(M\alpha+M_{O^{18}})}\frac{a_\alpha^2}{\hbar^2}\right]$; $a_\alpha=1.50(4^{1/3}+18^{1/3})\times10^{-13}$ cm $=6.31\times10^{-13}$ cm.

[c] $\theta_n^2=\gamma_n^2\left[\frac{2M_nM_{Ne^{21}}}{3(M_n+M_{Ne^{21}})}\frac{a_n^2}{\hbar^2}\right]$; $a_n=1.50(1^{1/3}+21^{1/3})\times10^{-13}$ cm $=5.64\times10^{-13}$ cm.

The quantities ω_l and ϕ_l are given by

$$\omega_l=\sum_{n=1}^{l}\tan^{-1}(\eta_\alpha/n)\quad\text{for}\quad l\geq1\,,\quad \omega_0=0\,,\tag{5}$$

$$\phi_l=\tan^{-1}(F_l/G_l)\,,\tag{6}$$

where F_l and G_l are, respectively, the regular and irregular Coulomb wave functions, and are functions of energy through the parameter $\rho_\alpha=k_\alpha a_\alpha$, where a_α is the interaction radius parameter.

The resonance parameters for levels 1 and 2 are given by

$$\begin{aligned}A_{11}&=(E_2-E+\Delta_2-\tfrac{1}{2}i\Gamma_2)/D\,,\\A_{22}&=(E_1-E+\Delta_1-\tfrac{1}{2}i\Gamma_1)/D\,,\\A_{12}&=(-\Delta_{12}+\tfrac{1}{2}i\Gamma_{12})/D\,,\end{aligned}\tag{7}$$

where

$$D=(E_1-E+\Delta_1-\tfrac{1}{2}i\Gamma_1)(E_2-E+\Delta_2-\tfrac{1}{2}i\Gamma_2)-(-\Delta_{12}+\tfrac{1}{2}i\Gamma_{12})^2.\tag{8}$$

In the present experiment the interfering levels were either $J^\pi=0^+$ or 1^-, and the exit channels were for alpha-particle emission or neutron emission, so that the widths in (7) and (8) are given by

$$\Gamma_{ij}=\frac{2\rho_\alpha}{F_l^2+G_l^2}(\gamma_{\alpha il}^2\gamma_{\alpha jl}^2)^{1/2}+2P_{nl}(\gamma_{nil_n}^2\gamma_{njl_n}^2)^{1/2}\,,\tag{9}$$

with $i=1$ or 2, $\Gamma_{11}\equiv\Gamma_1$, $\Gamma_{22}\equiv\Gamma_2$, and $\Gamma_{12}=\Gamma_{21}$. The quantity $\gamma_{nil_n}^2$ is the reduced neutron width of level i; $\gamma_{\alpha il}^2$ is the reduced alpha-particle width of level i; and P_{nl} is ρ_n for $l_n=0$ neutrons, is $\rho_n/[1+1/\rho_n^2]$ for $l_n=1$ neutrons, and is $\rho_n/[(3/\rho_n^2-1)^2+9/\rho_n^2]$ for $l_n=2$ neutrons. The subscript l_n indicates that the orbital angular momentum of the emitted neutrons is not necessarily the same as that of the emitted alpha particles. It is probable that most of the neutrons emitted by Ne^{22} will go to the ground state of Ne^{21} which is[9] $\frac{3}{2}^+$, or to the 350-keV first excited state which is[9] $\frac{5}{2}^+$. Thus, if $l=J=0$, 1, or 2, the lowest l_n values would be 2, 1, and 0, respectively.

In the approximation used in the derivation of Eq. (1) (namely, that the matrix $\mathbf{R}^0\,\mathbf{L}^0$ is zero), the level shifts become

$$\Delta_{ij}=-\sum_{c=\alpha}^{n}(S_c-B_c)(\gamma_{cil_c}^2\gamma_{cjl_c}^2)^{1/2}\,,\tag{10}$$

where $\Delta_{11}\equiv\Delta_1$, $\Delta_{22}\equiv\Delta_2$, $\Delta_{12}=\Delta_{21}$. The quantity B_c is the channel radius a_c multiplied by the logarithmic derivative of the wave function on the channel surface. The shift factor S_c is defined as

$$S_c=\rho_c(d/d\rho_c)\ln(F_c^2+G_c^2)^{1/2}\,,\tag{11}$$

where, for neutrons, the Coulomb wave functions are replaced by spherical Bessel functions:

$$F_c^2+G_c^2\rightarrow\rho_n^2[j_{l_n}^2(\rho_n)+n_{l_n}^2(\rho_n)]\,.\tag{12}$$

The quantity ρ_n is obtained from the reaction kinematics and is

$$\begin{aligned}\rho_n=k_na_n&=[(2M_nE_n)^{1/2}/\hbar]a_n\\&=0.730R_0[E_\alpha^{\text{Lab}}(\text{MeV})-0.862]^{1/2}.\end{aligned}\tag{13}$$

Both Yagi and Lane and Thomas take $S_c=B_c$ in their derivation of the two-level formula so that $\Delta_1=\Delta_2=\Delta_{12}=0$. Such an approximation should be valid if S_c is reasonably constant in the energy region of interest.

The two-level formula, Eq. (1), was used in the simultaneous analysis of the levels at 2.56 and 2.72 MeV and of the levels at 2.90 and 3.17 MeV. The formula was also used for two levels at 3.55 and 3.60 MeV which were outside the range of the elastic scattering measurements, but whose effect on the scattering measurements was quite pronounced in the vicinity of 3.50 MeV. This effect will be discussed later.

[9] T. Lauritsen and F. Ajzenberg-Selove, in *Nuclear Data Sheets,* compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.), NRC 61–5,6–3.

One-Level Formula

In the single-level approximation, the elastic scattering cross section of spinless particles by spinless nuclei is given by

$$\frac{d\sigma(\theta)}{d\Omega}=\lambda_\alpha{}^2\Bigg[-(\pi)^{1/2}C(\theta)+\frac{i}{2}\sum_{l'=0}^{\infty}(2l'+1)P_{l'}(\cos\theta)[e^{2i\omega_{l'}}-e^{2i(\omega_{l'}-\varphi_{l'})}]+\sum_{\{l \text{ for all levels}\}}\frac{(2l+1)P_l(\cos\theta)\rho_\alpha e^{2i(\omega_l-\varphi_l)}\gamma_{\alpha l}{}^2}{(F_l{}^2+G_l{}^2)(E_\lambda-E+\Delta_\lambda-i\Gamma/2)}\Bigg]^2, \quad (14)$$

where Γ is the total width of the resonance, Δ_λ is the level shift, E_λ is the "characteristic energy," and the remaining parameters are identical to those used in the two-level formula. Equation (14) was used in the analysis of the anomalies at $E_\alpha{}^{\rm Lab}=2.48$ and 3.33 MeV, and was also used to make preliminary fits to all the other resonances.

Computer Program

An IBM-1620 computer program was designed as follows: From the tabulated curves of Sharp, Gove, and Paul,[10] ten values of ϕ_l were obtained at approximately evenly spaced energy intervals in the region $E_\alpha{}^{\rm Lab}=2.40$ to 3.50 MeV for $R_0=1.5$ F (used throughout the analysis) and $l=0$, 1, 2, 3, and 4. The computer was then used to calculate the best nth-degree polynomial fit to the points by the method of least squares. These "computer curves" of ϕ_l were then used to calculate the first two terms in Eqs. (1) and (14) over the entire energy region for the five scattering angles, where $l=0$ to $l=4$ was used since ϕ_l was negligible for $l\geq 5$. The experimental data were then normalized at $E_\alpha{}^{\rm Lab}=2.40$ MeV to the cross sections calculated by the computer.

Curves of $F_l{}^2+G_l{}^2$ versus $E_\alpha{}^{\rm Lab}$ were obtained by least-squares fits similar to those of ϕ_l versus $E_\alpha{}^{\rm Lab}$. The experimental data indicated that only l values (i.e., J values) of 0, 1, and 2 were needed. The derivatives of these curves with respect to ρ_α were also included in the program in order to calculate the alpha-particle level shift $\Delta_{\lambda\alpha}$. Derivatives of the spherical Bessel functions of orders 2, 1, and 0 were also incorporated into the program to calculate the neutron level shift $\Delta_{\lambda n}$. The program was designed to obtain $(d\sigma/d\Omega)^{\rm c.m.}$ as a function of $E_\alpha{}^{\rm Lab}$; there were three adjustable parameters for each scattering anomaly, namely, E_λ, $\gamma_{\lambda\alpha l}{}^2$, and $\gamma_{\lambda n l_n}{}^2$. By working with the amplitudes in Eqs. (1) and (14), one can add the real and imaginary parts of the nuclear contributions from the various levels to those of the Rutherford and potential scattering contributions, and then simply take the sum of the squares of the resultant to obtain the cross section.

Discussion

The total cross section for the reaction $O^{18}(\alpha,n)Ne^{21}$ at resonance is given by $\sigma=(2J+1)4\pi\lambda_\alpha{}^2\Gamma_\alpha(\Gamma-\Gamma_\alpha)/\Gamma^2$. It is a simple matter to solve this quadratic equation for Γ_α since J is determined from the $O^{18}(\alpha,\alpha)O^{18}$ experiment and σ, Γ, and E_λ were measured in the $O^{18}(\alpha,n)Ne^{21}$ experiment. The solution of the quadratic equation gives two values of Γ_α; the proper value was selected on the basis of preliminary attempts at fitting the scattering data using a desk calculator. Once the appropriate Γ_α is known, the reduced width parameter $\gamma_{\alpha l}{}^2=(F_l{}^2+G_l{}^2)\times\Gamma_\alpha/2\rho_\alpha$ can be obtained at the resonance energy of the anomaly. The parameter $\gamma_{nl_n}{}^2$ can be obtained in a similar manner from $\Gamma_n=\Gamma-\Gamma_\alpha$. Estimates of $\gamma_{\alpha l}{}^2$, $\gamma_{nl_n}{}^2$, E_λ, and Δ_λ were made for each of the six observed anomalies in the energy region 2.40 to 3.50 MeV. A preliminary attempt was made to fit each of the observed six levels in Table I in the single-level approximation by including the energy variation of Δ_λ, Γ, Γ_α, and Γ_n. All six levels were allowed to interfere together, and the parameters were readjusted to obtain the best fits consistent with the experimental data. Fair qualitative agreement was obtained at the low-energy end, but poor agreement was found at the upper end, except at 90°. The behavior in the vicinity of 3.50 MeV was suggestive of an interfering level of $J^\pi=1^-$ at some energy above 3.50 MeV, and the level at 3.60 MeV [observed in the $O^{18}(\alpha,n)Ne^{21}$ experiment] appeared to be a good prospect. It was found that by making reasonable estimates of the parameters of this level that the agreement between theory and experiment was considerably improved.

There was one notable exception to these theoretical fits, which occurred at level 5 at 3.17 MeV. The theoretical cross sections were consistently higher than the experimental points in the neighborhood of 3.10 MeV. Several attempts were made to correct this situation: (1) Using $J^\pi=0^+$, the total width of level 5 was increased in varying steps from 80 to 115 keV; the resonance was made broader by this procedure, but the peak-to-valley ratio diminished. (2) For a given Γ, a larger Γ_α than predicted by the $O^{18}(\alpha,n)Ne^{21}$ data was

[10] W. T. Sharp, H. E. Gove, and E. B. Paul, Atomic Energy of Canada Limited Report, AECL-268, 1953 (unpublished). These curves were derived from the tables of I. Block, M. M. Hull, A. A. Broyles, W. G. Bouricius, B. E. Freeman, and G. Breit, Rev. Mod. Phys. **23**, 147 (1951).

chosen. By varying Γ_α it was found that a peak-to-valley ratio consistent with the scattering data could be obtained, but the theoretical curve was still consistently higher than the experimental points. In addition, the slope of the theoretical curve was much greater than the slope of a curve drawn by eye through the experimental points at all angles except 90°. (3) The assumption was then made that the level could be $J^\pi=4^+$ rather than $J^\pi=0^+$. From the $O^{18}(\alpha,n)Ne^{21}$ data a reduced alpha-particle width of reasonable magnitude could be calculated, but the reduced neutron width exceeded the Wigner limit by a factor of 2. If the roles of alpha particle and neutron widths are interchanged in order to obtain a reasonable neutron reduced width, then the alpha-particle reduced width exceeds the Wigner limit. Another reason for discarding a $J^\pi=4^+$ assignment is based on the rather large observed dip at 152.3°. If the level truly were 4^+, there would be no resonance effect in the cross section at $\theta_{\text{c.m.}}=149.4°$. Since 149.4° is so close to 152.3°, it is unlikely that the large experimental dip is due to a $J^\pi=4^+$ level. (4) Another possibility is that there is a broad $J^\pi=0^+$ level at an energy above 3.50 MeV that is causing the difficulty in the vicinity of 3.17 MeV. Such a level would have to be 0^+ to explain the behavior at 3.17 MeV, but would likely cause trouble at 90° in the vicinity of 3.50 MeV. Several estimates of the level parameters consistent with the $O^{18}(\alpha,n)Ne^{21}$ data were made, but all attempts proved futile in accounting for the behavior of level 5. (5) Finally, a broad low-lying $J^\pi=0^+$ level in the vicinity of 3.17 MeV was analyzed together with level 5 in the two-level approximation to see if the behavior of level 5 could be accounted for. This attempt proved as futile as the other four attempts.

A better fit to the data (levels 1–6) is plotted in Figs. 1 and 2 as the solid line. Levels 1 and 6 were analyzed in the single-level approximation; levels 2 and 3 and levels 4 and 5 were analyzed in the two-level approximation. In the two-level approximation, the assumption was made that $\Delta_1=\Delta_2=\Delta_{12}=0$. This assumption implies that $S_c=B_c$ in Eq. (10). The parameters used to obtain the solid line are tabulated for levels 1–6 in Table I. [It should be pointed out that the values given in Table I have been adjusted to fit the scattering data, and hence do not give the particular values obtained from the $O^{18}(\alpha,n)Ne^{21}$ experiment.]

In order to account for the fall-off in cross section in the vicinity of 3.50 MeV, a $J^\pi=1^-$ level at 3.60 MeV was postulated. It was found by adjusting the parameters of this level that the fall-off in the vicinity of 3.50 MeV was roughly half-way between the solid and dashed curves of Figs. 1 and 2. Another level ($J^\pi=1^-$) was postulated at 3.55 MeV, and the combined levels 7 and 8 at 3.55 and 3.60 MeV were analyzed simultaneously in the two-level approximation. The result of this last calculation is plotted in Figs. 1 and 2 as the dashed curve; the assumed parameters of levels 7 and 8 are tabulated in parentheses in Table I.

Finally, it might be conjectured that the behavior between 2.80 and 3.20 MeV is due to the combined effect of the interference between levels 4 and 5 and broad $J^\pi=0^+$ levels at $E_\alpha^{\text{Lab}}>3.50$ MeV and in the vicinity of 3.10 MeV. The proper analysis of these three or four interfering levels of the same spin and parity should appropriately be carried out in the three- or four-level approximation. Such formulas could be developed from the treatment by Lane and Thomas. A four-level formula would involve ten resonance terms instead of three in Eq. (1) and would be considerably more complicated due to the increased complexity of D in Eq. (8). In addition, it would appear necessary to extend the $O^{18}(\alpha,\alpha)O^{18}$ measurements above 3.50 MeV in order to better determine the effects of higher lying levels.

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